6.3) a) $Q(x) = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_1x_3 - 2x_2x_3$

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$ simétrica tq $Q(x) = x^T A x$

Busco autovalores y autovectores:

$$P(\lambda)=\det\left(\begin{bmatrix}\lambda-2 & 1 & 1\\ 1 & \lambda-2 & 1\\ 1 & 1 & \lambda-2\end{bmatrix}\right)=(\lambda-2)\cdot\begin{vmatrix}\lambda-2 & 1\\ 1 & \lambda-2\end{vmatrix}-\begin{vmatrix}1 & 1\\ 1 & \lambda-2\end{vmatrix}+\begin{vmatrix}1 & 1\\ \lambda-2 & 1\end{vmatrix}=$$

$$=(\lambda-2)\cdot(\lambda^2-4\lambda+3)-(\lambda-2-1)+(1-\lambda+2)$$

$$=\lambda^3-4\lambda^2+3\lambda-2\lambda^2+8\lambda\cancel{-6}-\lambda\cancel{+3+3}-\lambda$$

$$=\lambda^3-6\lambda^2+9\lambda=\lambda\cdot\underbrace{(\lambda^2-6\lambda+9)}_{\frac{6\pm\sqrt{36-36}}{2}}$$

Autovalores: $\boxed{\begin{matrix}\lambda_1=0\\ \lambda_2=3\\ \lambda_3=3\end{matrix}}$

<u>Para $\lambda=3$</u>

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}\qquad x=-y-z$$

$$\rightarrow \bar{x}=(-y-z,\,y,\,z)=y\cdot\underbrace{\overset{U1}{(-1,1,0)}}+z\cdot\underbrace{\overset{U2}{(-1,0,1)}}$$

AVECT.
$\lambda=3$

Ortogonalizo por GS ~~a~~ $U1$ y $U2$:

~~[illegible]~~ $v1 = u1 = (-1,1,0)$

$$v2 = u2 - \frac{\langle u2, v1\rangle}{\langle v1, v1\rangle} \cdot v1$$

$$\rightarrow v2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - \frac{\left(\begin{bmatrix} -1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right)}{\begin{bmatrix} -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}} \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$\rightarrow v_2 = \begin{bmatrix}-1\\0\\1\end{bmatrix} - \left(\frac{1}{2}\cdot\begin{bmatrix}-1\\1\\0\end{bmatrix}\right)$

$\boxed{\rightarrow v_2 = \begin{bmatrix}-1\\0\\1\end{bmatrix} - \begin{bmatrix}-\frac{1}{2}\\\frac{1}{2}\\0\end{bmatrix} = \begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\1\end{bmatrix}}$

$\rightarrow$ ~~Lo~~ Los normalizo:

$v_1 = \begin{bmatrix}\frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\end{bmatrix}^T$

AUX

$\begin{bmatrix}-\frac{1}{2} & -\frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\1\end{bmatrix} = \frac{1}{4}+\frac{1}{4}+1 = \frac{1}{2}+1 = \frac{3}{2}$

$v_2 = \begin{bmatrix}\frac{-\sqrt{2}}{2\sqrt{3}} & \frac{-\sqrt{2}}{2\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}}\end{bmatrix}$

Ahora busco v_3:

Para $\lambda = 0$

$\cancel{\begin{pmatrix}-1 & 1 & 1\\1 & -1 & 1\\-1 & 1 & -1\end{pmatrix} \begin{matrix}F2 \to F1+F2\\F3 \to F1+F3\end{matrix} \begin{pmatrix}-1 & 1 & 1\\0 & 0 & 2\\0 & 2 & 0\end{pmatrix} \sim \begin{pmatrix}-1 & 1 & 1\\0 & 2 & 0\\0 & 0 & 2\end{pmatrix} \quad -x+y+z=0 \to x=0,\ y=0,\ z=0}$

$\begin{pmatrix}-2 & 1 & 1\\1 & -2 & 1\\1 & 1 & -2\end{pmatrix} \begin{matrix}F2 \to F1+2F2\\F3 \to F1+2F3\end{matrix} \begin{pmatrix}-2 & 1 & 1\\0 & -3 & 3\\0 & 3 & -3\end{pmatrix} F3 \to F2+F3 \begin{pmatrix}-2 & 1 & 1\\0 & -3 & 3\\0 & 0 & 0\end{pmatrix}$

$-2x+y+z=0 \rightarrow -2x+2y=0 \rightarrow y=x$

$\cancel{-3}y = \cancel{-3}z \rightarrow y=z$

$\rightarrow \vec{X} = y\cdot\underbrace{(1,1,1)}_{\substack{\text{AVECT}\\ \lambda=0}}$ Lo normalizo $\rightarrow v_3 = \begin{bmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{bmatrix}$

Por lo que puedo expresar A como:

$$A = \underbrace{\begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{\sqrt{2}}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-\sqrt{2}}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}}_{P} \cdot \underbrace{\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\Lambda} \underbrace{\begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-\sqrt{2}}{2\sqrt{3}} & \frac{-\sqrt{2}}{2\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}}_{P^T}$$

Con esa P, hago cambio de variable $x = Py$

donde $$\boxed{\check{Q}(y) = y^T \cdot \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot y = 3y_1^2 + 3y_2^2}$$

Forma cuadrática
sin prod. cruzados